ELECTRICAL MODELING OF NONSTEADY THERMAL PROCESSES IN MULTILAYER SANDWICH WALLS

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A method is discussed for the design of electrical models constructed of RC cells to study the nonsteady thermal processes in multilayer sandwich walls.

Determining the nonsteady temperature field in a multilayer sandwich wall is accomplished quite effectively by the method of electrical modeling with grid models. Here we can use RC networks [1, 3], R networks [2], and the like.

From the standpoint of speeding up calculations, particular interest has been expressed in models of RC networks. Basic relationships have been derived in [1, 3] which enable us to determine the parameters of an electrical model for the solution of various engineering problems. However, the direct application of these relationships to the modeling of certain boundary-value problems is difficult in a number of cases. These difficulties arise, in particular, in the modeling of nonsteady temperature fields in multilayer sandwich walls. In connection with the points raised in this paper, we examine a method of designing RC models for sandwich walls on the assumption that the time and temperature scales for the various layers of the sandwich wall are identical. For the model we employed a circuit consisting of m groups of series-connected RC cells (see Fig. 1).

We know from [3, 4] that a one-dimensional nonsteady thermal process in a plane sandwich wall and the transient electrical response in the chosen model are described by the following system of dimensionless equations:

$$\frac{\partial \psi_i}{\partial t} = A_{i,1} \frac{\partial^2 \psi_i}{\partial l_i^2} \quad \text{for} \quad 0 \leqslant l_i \leqslant L_i;\ t > 0,\ i = 1, 2, \ldots, m;$$

$$\left.\begin{aligned} \psi_i &= \psi_{i+1} \\ A_{i,2} \frac{\partial \psi_i}{\partial l_i} &= \frac{\partial \psi_{i+1}}{\partial l_{i+1}} \end{aligned}\right\} \quad \text{for} \quad l_i = L_i;\ l_{i+1} = 0;\ t > 0,\ i = 1, 2, \ldots, (m-1); \tag{1}$$

$$\frac{\partial \psi_1}{\partial l_1} + A_l (\psi_l - \psi_1) = 0 \quad \text{for} \quad l_1 = 0 \text{ and } t > 0;$$

$$\frac{\partial \psi_m}{\partial l_m} + A_r (\psi_m - \psi_r) = 0 \quad \text{for} \quad l_m = L_m \text{ and } t > 0;$$

$$\psi_i = \psi_{in} \quad \text{for} \quad t = 0.$$

Here we make the following assumptions: 1) the wall layers are in close contact with each other and perfect thermal contact exists between them; 2) the thermophysical characteristics of the layer materials are constant and equal to the average values for the working temperature range; 3) the initial temperature at all points and layers of the wall is constant and equal to T_{in}.

In the system of equations (1) for the thermal process the dimensionless complexes and simplexes have the following values:

$$A_{i,1_t} = \frac{a_i \tau^*}{(x_i^*)^2}; \quad A_{i,2_t} = \frac{\lambda_i x_{i+1}^*}{\lambda_{i+1} x_i^*}; \quad A_{l_t} = \frac{\alpha_t x_1^*}{\lambda_1}; \quad A_{r_t} = \frac{\alpha_r x_m^*}{\lambda_m};$$

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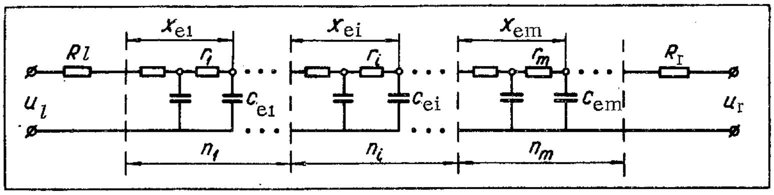

Fig. 1. Circuit diagram of the electrical model of a multilayer sandwich wall.

$\psi_i = \theta_i = T_i/T^*$ is the instantaneous relative temperature; $\psi_l = \theta_l = T_l/T^*$ and $\psi_r = \theta_r = T_r/T^*$ are the relative temperatures of the ambient media; $L_i = \delta_i/x_i$ is the relative thickness of the wall layer, where T^*, x^*, and τ^* are the so-called reference values of the temperature T, the coordinate x, and the time τ.

The dimensionless complexes and simplexes pertaining to the electrical process have the following values:

$$A_{i,1\text{e}} = \frac{\tau_\text{e}^*}{r_i c_{\text{e},1}(x_{\text{e},1}^*)^2}; \quad A_{i,2\text{e}} = \frac{r_{i+1}x_{\text{e},i+1}^*}{r_i x_{\text{e},i}^*};$$

$$A_{l_\text{e}} = \frac{r_1 x_{\text{e},1}^*}{R_l}; \quad A_{\text{r}_\text{e}} = \frac{r_m x_{\text{e},m}^*}{R_\text{r}};$$

$\psi_i = U_i = u_i/u^*$ is the instantaneous relative voltage; $\psi_l = U_l = u_l/u^*$ and $\psi_r = U_r = u_r/u^*$ are the relative supply voltages; $L_i = n_i/x_{\text{el},i}^*$ is the relative number of cells in the given layer, where u^*, x_{el}^*, and τ_{el}^* are the reference values of the voltage u, of the coordinate x_{el}, and of the time τ_{el}.

We can take any pair of appropriate similar parameters [5] as the reference values for T^*, x^*, τ^* and u^*, x_{el}^*, τ_{el}^*. We will take the following quantities as the reference values: T_l, δ_i, τ_c and u_l, n_i, $\tau_{\text{el,c}}$.

The quantitative relationships between the thermal and electrical quantities can be determined from the condition of identity for the corresponding generalized equations of the thermal and electrical processes.

We use the following notation:

$$k_t = \frac{T^*}{u^*} = \frac{T_l}{u_l} \text{ is the temperature scale,} \tag{2}$$

$$k_{l,i} = \frac{x_i^*}{x_{\text{e},i}^*} = \frac{\delta_i}{n_i} \text{ is the coordinate scale,} \tag{3}$$

$$k_\tau = \frac{\tau^*}{\tau_\text{e}^*} = \frac{\tau_c}{\tau_{\text{e,c}}} \text{ is the time scale.} \tag{4}$$

We should stress that the scales k_t and k_τ are assumed to be identical for the various layers. The coordinate scale k_l may vary for these various layers. The use of various scales with respect to the coordinate substantially expands the possibilities of using an electrical model.

Bearing in mind that when the similar generalized equations of the thermal and electrical processes are equal the relative quantities θ, l_i, and t will, respectively, equal the quantities U, $l_{\text{el},i}$, and t_{el}, we have:

$$T = k_T u, \tag{5}$$

$$x_i = k_{l,i} x_{\text{e},i}, \tag{6}$$

$$\tau = k_\tau \tau_\text{e}. \tag{7}$$

When the corresponding dimensionless complexes – after simple transformations – are equal, we obtain the following basic equations for the design of electric models:

$$k_\tau = \frac{\delta_i^2}{a_i r_i c_{\text{e},i} n_i^2}, \tag{8}$$

$$\frac{\delta_i}{\lambda_i r_i n_i} = \frac{1}{\alpha_l R_l} = \frac{1}{\alpha_r R_r} = \text{const}, \tag{9}$$

TABLE 1. Variants for the Design of the First and i-th Model Layers

Calculation variant	Specified	Determined
	First layer	
1	r_1 ; $c_{e,1}$; k_τ	n_1 ; $k_{l,1}$; k_t
2	r_1 ; n_1 ; k_τ	$c_{e,1}$; $k_{l,1}$; k_t
3	$c_{e,1}$; n_1 ; k_τ	r_1; $k_{l,1}$; k_t
4	r_1 ; $c_{e,1}$; n_1	k_τ; $k_{l,1}$; k_t
	i-th layer	
1	r_i	$c_{e,i}$; n_i; $k_{l,i}$
2	n_i	$c_{e,i}$; r_i; $k_{l,i}$
3	$c_{e,i}$	n_i; r_i; $k_{l,i}$

$$\frac{\delta_i c_i \gamma_i}{n_i c_{e,i}} = \text{const.} \tag{10}$$

The design of the model is best begun by calculating the electrical parameters for the first group of cells.

Thus we have to calculate r_1, $c_{e,1}$, n_1, and k_τ. For this we can use function (8). This gives us four unknowns and one theoretical relationship. Consequently, for a uniquely defined solution we have to assume three quantities. The number of possible means of calculating the parameters for the first group of cells is defined as the number of possible combinations of three elements from among four elements, i.e., four.

For the i-th group of cells, with the exception of the first, we have to determine the quantities r_i, $c_{e,i}$, and n_i. Here we can use the two relationships

$$\frac{\delta_i}{\lambda_i r_i n_i} = \frac{\delta_1}{\lambda_1 r_1 n_1};$$

$$\frac{\delta_i c_i \gamma_i}{n_i c_{e,i}} = \frac{\delta_1 c_1 \gamma_1}{n_1 c_{e,1}}.$$

The temperature scales k_t and the coordinate scales $k_{l,i}$ are calculated according to (2) and (3). All of the calculation variants are shown in Table 1.

The boundary resistance R_l and R_r are calculated according to the relationshps

$$R_l = \frac{\lambda_1 r_1 n_1}{\delta_1 \alpha_l}; \quad R_r = \frac{\alpha_l}{\alpha_r} R_l.$$

The selection of a given calculation variant is governed by the suitable and most convenient conditions for the fabrication and operation of the electrical model. Thus, for example, the coordinate scale k_l, as a rule, is not a decisive quantity, since it is a function of the number of electrical cells in the given "layer" of the model. The temperature scale k_t is also not a decisive quantity, since it depends on the supply voltage whose magnitude is controlled with comparative ease.

On the other hand, quite frequently we have to solve problems in the artificial time scale, and the scale k_τ is therefore frequently a decisive quantity.

In constructing an electrical model it is advisable to choose such a design variant and to calculate its basic parameters in such a manner that it is possible to employ electric-capacitance constants of standardized ratings in each "layer" of the model.

Proceeding from the above, we should regard those calculation variants which among the specified parameters include the electric capacitances and the time scale as the most rational from the standpoint of model design simplicity and the conditions of simulation.

Thus, on the basis of the relationships derived for m-layer walls we have developed a method for the design of models which reduces the design operation to an orderly system and eliminates the arbitrary approach in the selection of the parameters. We have established that in designing multilayer models we must employ the relationships which associate the capacitance and impedance of the various layers in the entire system. This imposes additional conditions on the modeling and imparts great methodological orderliness to the design. The proposed design method makes it possible to predetermine the values of some of the most desirable parameters of the model.

NOTATION

T	is the instantaneous temperature;
T_l, T_r	are the temperatures of the ambient medium, to the left and to the right of the wall, respectively;
λ, a	are the thermal conductivity and thermal diffusivity, respectively;
α_l, α_r	are, respectively, the heat-transfer coefficients for the left- and right-hand boundaries of the wall;
c	is the specific heat capacity;
γ	is the density;
τ	is the time;
x	is a coordinate;
δ	is the thickness of the wall layer;
θ, l, t	are the relative temperature, coordinate, and time, respectively;
u	is the instantaneous voltage;
u_l, u_r	are the supply voltages;
r, R	are the electrical resistances;
c_{el}	is the electrical capacitance;
x_{el}	is the coordinate of the electric circuit;
n	is the number of electric cells;
τ_{el}	is the duration of the electrical process;
U, l_{el}, t_{el}	are the relative voltage, coordinate, and time for the electric circuit;
τ_c, $\tau_{el,c}$	are the durations of the nonsteady thermal and transient electric processes.

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